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The Effect of FLR Correction on Rayleigh Taylor Instability of two Superposed Viscous Fluids in Porous Medium

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Abstract- *This paper is aimed to study the Rayleigh Taylor instability of two superposed incompressible viscous fluids in porous medium in presence of FLR (finite Larmor radius) correction. The problem is numerically solved using the normal mode analysis method. A dispersion relation of two uniform fluids of different densities separated by a common boundary is investigated by using appropriate boundary conditions. Numerical analysis is performed to show the effect of FLR and porosity on the growth rate of Rayleigh Taylor instability and it is found that the porosity and FLR correction have stabilizing influence on the growth rate of considered Rayleigh Taylor Configuration.*

Keywords: *Dynamic viscosity, FLR (finite Larmor radius), Porosity, Permeability,*

I. Introduction

Rayleigh Taylor instability is instability at the interface of two fluids of different densities that arises when denser fluid is supported by lighter fluid in presence of gravitational force. The study of R -T instability is imperative because it has relevance in nuclear fusion research, space physics and astrophysics. It plays crucial role in ICF (inertial confinement fusion) and important for nuclear fusion. It is also important in supernova implosion, explosion, crab nebula, and ionospheric irregularities. Chandrasekhar [1] has discussed R -T instability in detailed in their monograph. Development of R -T instability is extended by Menikoff [2], Mikaelian [3] taking various assumption in non porous medium. Sanghvi and Chhajlani [4] have discussed combined effect of suspended particle and finite Larmor radius on the R-T instability. Sharma and Chhajlani [5,6] have studied the rotation effect with FLR correction on R-T instability. El-Sayed [7] has carried out the investigation of FLR correction on R -T instability of superposed viscous fluids. Recently, the consequence of viscosity and suspended particles on R-T instability is also studied by Sharma et al. [8].

The problem of R-T instability in porous medium has great importance in geophysics, astrophysics and space physics. From this point of view authors and researchers have taken into account the R-T instability of fluids in porous medium. Sharma and Spanos [9] have discussed the instability of streaming fluids in porous medium. Sunil and Chand [10] have solved the R-T instability with effect of suspended particle and variable magnetic field in porous medium. Recently Prajapati and Chhajlani [11] have investigated the effect of suspended particles and surface tension on K-H and R-T instability. More recently Singh and Dixit [12] have studied R-T instability of two superposed viscoelastic fluids with effect of rotation and suspended particles. Kango[13] also conferred the R-T instability of two superposed viscoelastic fluids in porous medium.

In present study we wish to find out the effect of FLR correction and suspended particles on Rayleigh Taylor instability of two superposed viscous fluids in porous medium.

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II. Formulation of Problem

We consider a problem of two semi infinite non dissipative, homogeneous, incompressible viscous superposed magneto fluids separated by a plane interface z=0 in porous medium. It is supposed that suspended particles of uniform size and spherical shape are permeated with fluids homogeneously and exert a stoke drag force KN(V-U) on the fluid where, N is density of suspended particles and K is a constant given as K =6πaμ.The fluid is under the action of magnetic field **H**(H_x, 0, 0) in X direction and gravitational force **g**(0,0,-g) in Z direction. The resistance force [-(μ/k_1) u] describes the macroscopic properties of unsteady flow in porous medium.Therefore the appropriate linearized basic equations of the problem are

$$\frac{\rho}{\varepsilon}\frac{\partial \mathbf{u}}{\partial t} = -\nabla\delta p - \nabla.\vec{\mathbf{P}} + \delta\rho\mathbf{g} + \frac{KN(\mathbf{v}-\mathbf{u})}{\varepsilon} - \frac{\mu}{k_1}\mathbf{u} + \frac{\mu_e}{4\pi}(\nabla\times\mathbf{h})\times\mathbf{H} + \frac{\mu\nabla^2\mathbf{u}}{\varepsilon} + \frac{1}{\varepsilon}\left(\frac{\partial w}{\partial \mathbf{X}} + \frac{\partial \mathbf{u}}{\partial z}\right)\frac{\partial\mu}{\partial z} \tag{1}$$

$$\varepsilon\frac{\partial\delta\rho}{\partial t} + (\mathbf{u}.\nabla)\rho = 0 \tag{2}$$

$$\nabla.\mathbf{u} = 0 \tag{3}$$

$$\nabla.\mathbf{h} = 0 \tag{4}$$

$$\varepsilon\frac{\partial \mathbf{h}}{\partial t} = (\mathbf{H}.\nabla)\mathbf{u} \tag{5}$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\mathbf{v} = \mathbf{u} \tag{6}$$

Here the symbols ρ, μ, p, μ_e denotes fluid density, viscosity, fluid pressure and permeability of magnetic field respectively and δp, δρ, **u** (u,v,w), v, **h** (h_x,h_y,h_z), represents the perturbation in pressure, fluid density, velocity of fluid, velocity of suspended particle, and magnetic field respectively. The relaxation time for the suspended particles is denoted by τ = m/k.

The stress tensor $\vec{\mathbf{P}}$ has the following components for the horizontal magnetic field **H** (H_x, 0, 0)

$$\begin{aligned} P_{xx} &= 0 \\ P_{yy} &= -P_{zz} = -\rho\nu_0\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \\ P_{xy} &= P_{yx} = -2\rho\nu_0\left(\frac{\partial u}{\partial z}\right) \\ P_{xz} &= P_{zx} = 2\rho\nu_0\left(\frac{\partial u}{\partial y}\right) \\ P_{zy} &= P_{yz} = \rho\nu_0\left(\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z}\right) \end{aligned} \tag{7}$$

Here the parameter ν_0 is taken as $\nu_0 = \frac{R_L^2\Omega_L}{4}$ where R_L is finite Larmor radius and Ω_L is ion gyro frequency.

To investigate stability of the system, we take space and time dependent perturbation equation which represents propagation of waves. Here k is wave number along y axis and (*i n)* is growth rate of the perturbation.

$$\exp(iky + int) \tag{8}$$

By using and solving equations (1) to (8), we obtain the general solution of the considered problem as

$$\left[\frac{in}{\varepsilon} + \frac{\upsilon}{k_1} + \frac{\alpha in}{\varepsilon[\tau in + 1]}\right]\left[D(\rho Dw) - k^2\rho w\right] + 2i\nu_0 k\left[D(D\rho\, Dw) - k^2(D\rho)w\right] + \frac{gk^2 w(D\rho)}{\varepsilon in} - \left[\begin{array}{l} D\left\{\frac{\mu}{\varepsilon}\left(D^2 - k^2\right)Dw\right\} - \\ \frac{k^2\mu}{\varepsilon}\left(D^2 - k^2\right)w \end{array}\right] - \left[D\left\{\begin{array}{l}\frac{D\mu\left(D^2 - k^2\right)w}{\varepsilon} - \\ \frac{2k^2 D\mu\, Dw}{\varepsilon}\end{array}\right\}\right] = 0$$

(9)

Where α = mN/ρ is mass concentration of the suspended particle, υ = μ/ρ is kinematic viscosity and D is short form of d/dz.

Equation (9) represents general differential equation including the effect of FLR correction, suspended particles and viscosity in porous medium.

III. Dispersion Relation

We consider the case of two superposed fluids of different densities ρ_1 (z < 0), ρ_2 (z > 0) and different viscosities μ_1 (z < 0), μ_2 (z > 0) separated by a plane interface z = 0. Thus, in this region at constant density ρ and viscosity μ, equation (9) becomes

$$\left(D^2 - k^2\right)\left(D^2 - q_j^2\right)w = 0 \qquad (10)$$

Where

$$q_j^2 = k^2 + \frac{1}{\upsilon_j}\left[in + \frac{\varepsilon\upsilon_j}{k_1} + \frac{\alpha_j in}{\tau in + 1}\right] \qquad (11)$$

j = 1, 2

the general solution of equation (10) becomes

$$w_1 = A_1 \exp(kz) + B_1 \exp(q_1 z) \qquad (12)$$

At (z < 0)

$$w_2 = A_2 \exp(-kz) + B_2 \exp(-q_2 z) \qquad (13)$$

At (z > 0)

Where A_1, B_1, A_2, B_2 are arbitrary constant.

The above solution must satisfy boundary condition at the interface z = 0

1. $w_1 = w_2$

2. $Dw_1 = Dw_2$

3. $\mu_1\left(D^2 + k^2\right)w_1 = \mu_2\left(D^2 + k^2\right)w_2$

4. The total pressure should be continuous at the interface z = 0

The condition (4) is satisfied by integrating equation (9) across the interface as follow s

$$\begin{aligned}&\Delta_0\left[\rho Dw\left\{\frac{in}{\varepsilon} + \frac{\upsilon}{k_1} + \frac{\alpha in}{\varepsilon[\tau in + 1]}\right\}\right] \\ &- 2ikv_0 k^3 \Delta_0(\rho)w + \frac{gk^2 w \Delta_0(\rho)}{\varepsilon in} \\ &- \Delta_0\left[\frac{\mu\left(D^2 - k^2\right)w}{\varepsilon}\right] + \\ &\frac{2k^2 \Delta_0(\mu) Dw}{\varepsilon} = 0\end{aligned} \qquad (14)$$

Applying the boundary conditions on the system of two superposed viscous fluids we obtain the following equation.

$$\begin{aligned}&c^3 + c^2\begin{bmatrix} f_0(1+\alpha') + \upsilon'\varepsilon\left(2k^2 + \frac{1}{k_1}\right) + \\ 2iv_0 k^2 f_0 \varepsilon(\beta_2 - \beta_1) \end{bmatrix} \\ &+ c\begin{bmatrix} f_0\upsilon'\varepsilon\left(2k^2 + \frac{1}{k_1}\right) + 2iv_0 k^2 f_0 \varepsilon(\beta_2 - \beta_1) \\ - gk(\beta_2 - \beta_1) \end{bmatrix} \\ &- gkf_0(\beta_2 - \beta_1) = 0\end{aligned} \qquad (15)$$

Where $c = in$ and $\tau = 1/f_0$, $\alpha' = \frac{2mn}{\rho_1 + \rho_2}$,

$\beta_1 = \frac{\rho_1}{\rho_1 + \rho_2}$, $\beta_2 = \frac{\rho_2}{\rho_1 + \rho_2}$, $\upsilon' = \frac{\mu}{\rho_1 + \rho_2}$

The equation (15) is the characteristic dispersion relation for two superposed viscous fluids of different densities with effect of suspended particles and finite Larmor radius correction in porous medium. Sanghvi and Chhajlani [4] have investigated this result in absence of porous medium for nonviscous fluids. El-Sayed [7] discussed this problem in non porous medium. Prajapati and Chhajlani [11] obtained dispersion relation with effect of surface tension in porous medium.

IV. Discussion

The Rayleigh Taylor instability of two superposed viscous fluids in porous medium is discussed for stable and unstable configuration by using Routh Hurwitz criterion.

IV.1 Stable Configuration ($\beta_1 > \beta_2$)

In this case we find that the constant term of equation (15) is positive and necessary condition of Routh Hurwitz criterion is fulfilled. This result indicates that the system of superposed viscous fluids in porous medium is stable.

IV.2 Unstable Configuration ($\beta_1 < \beta_2$)

In this case we find that the constant term of equation (15) is negative and the necessary condition of Routh Hurwitz criterion is not fulfilled. This indicates that the system has at least one real positive root which leads to the instability of system. We also find that FLR (finite Larmor radius), porosity,

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permeability, and dynamic viscosity do not take part in the condition of instability.

To study the impact of FLR (finite Larmor radius) and porosity on the growth rate of R-T instability we obtain dimensionless form of equation (14) by using following substitution

$$c^* = \frac{c}{\sqrt{gk}}, f_0^* = \frac{f_0}{\sqrt{gk}}, \upsilon^* = \frac{\upsilon'}{\sqrt[3]{gk}}, \nu_0^* = \frac{\nu_0}{\sqrt[3]{gk}}, k^* = k\sqrt{gk}, \frac{1}{k_1^*} = \frac{gk}{k_1}$$

$$c^{*3} + c^{*2}\left[\begin{array}{l} f_0^*(1+\alpha') + \upsilon'^*\,\varepsilon\left(2k^* + \dfrac{1}{k_1^*}\right) + \\ 2i\nu_0^* k^{*2}\,\varepsilon(\beta_2 - \beta_1) \end{array}\right] + c^*\left[\begin{array}{l} f_0^*\upsilon^*\varepsilon\left(2k^* + \dfrac{1}{k_1^*}\right) - (\beta_2 - \beta_1) + \\ 2i\nu_0^* k^{*2} f_0^*\varepsilon(\beta_2 - \beta_1) \end{array}\right] - f_0^*(\beta_2 - \beta_1) = 0 \qquad (16)$$

By taking numerical values of parameters $\alpha' = 0.35$, $\varepsilon = 0.2$, $\nu_0^* = 0.3, 0.5, 0.7, 0.9$, β_2-β_1=1.5, k_1^*=0.1, f_0^*=0.6, and υ'^*=0.5 we draw figure 1 between growth rate (c^*) and wave number (k^*). In this figure we found that FLR correction (, ν_0^*) has stabilizing influence on the growth rate of RT instability and figure 2 is depicted by putting values of parameters as $\alpha' = 0.35$, ε = 0.13, 0.26, 0.39 ν_0^* = 0.5, β_2-β_1=1.5, k_1^*=0.1, f_0^*=0.6 and υ'^*=1. We found that from figure 2 the growth rate is decreasing on increasing the value of porosity (ε) which indicates that porosity has stabilizing influence on the growth rate of RT instability. Therefore we conclude that R-T instability of system gets stabilized in presence of FLR correction and porous medium.

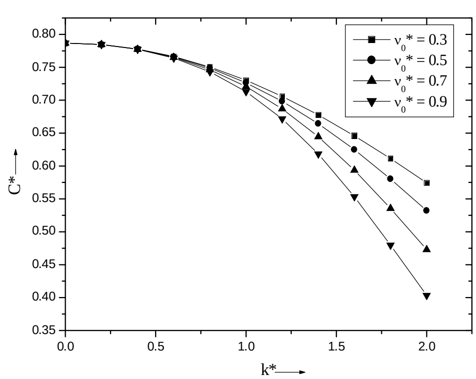

Fig 1 The growth rate C^* (of unstable R-T mode) verses wave number k* for variation of FLR (finite Larmor radius).

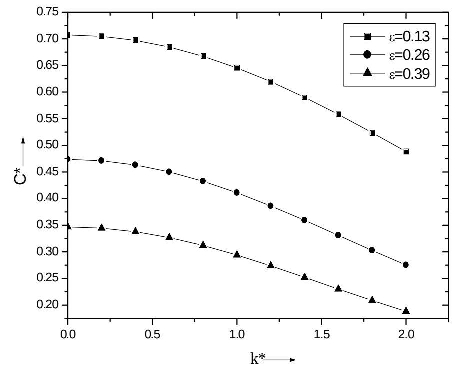

Fig.2 The growth rate C^* (of unstable R-T mode) verses wave number k* for variation of porosity of medium.

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References

[1] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic stability, Clarendon press, oxford*, 1961.

[2] R. Menikoff, R. C Mjolsness, D. H.Sharp, C. Zemach, *Unstable normal mode for Rayleigh Taylor instability in viscous fluids, Phys. Fluids, Vol. 20,* pp 2000,1977.

[3] K.O. Mikaelian, , *Phys. Rev. E Vol. 47*, pp 375, 1993.

[4] Superposed magnetized conducting plasma, *Phys. Plasmas, Vol. 5*, pp 2203, 1998.

[5] P.K. Sharma, R. K. Chhajlani, Z. Naurforsch., *Vol. 53a*, pp 937, 1998.

[6] M.F. El-Sayed, *Eur. Phys. J.D., Vol. 23*, pp 391, 2003.

[7] P. K. Sharma, R.P. Prajapati and R. K. Chhajlani, *Thermal Sci., Vol. 14*, pp 11, 2010.

[8] R. C. Sharma, T.J.T Spanos, *Can. J. Phys. Vol.60*, pp 1391, 1982.